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First Wall Magnetic Protection in an Inertially Confined Thermonuclear Reactor

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FIRST WALL MAGNETIC PROTECTION IN AN INERTIALLY CONFINED THERMONUCLEAR REACTOR

by

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ABSTRACT

On the basis of three different theories, it is shown for a typical set of reactor parameters that inertially confined micro-explosions are either

1. Stable during early expansion; or

2. Only weakly unstable during expansion with flute amplitude growth of but 0.005%; or

3. Only weakly unstable during expansion with negligible flute amplitude growth.

Simple formulas are given for skin depths in the plasma (~ 2.4 cm here) and in the first wall (~ 0.3 cm for 500°C graphite). Plasma behavior is found to be collective and ambipolar. Formulas are given for calculating the strength of the required protecting initial magnetic field. For a microexplosion putting 16-MJ kinetic energy into 0.25 g of lead debris, we find 3.2 kG to be adequate protection in a reactor chamber of 200-cm radius with a graphite (or other conducting) wall.

I. INTRODUCTION

This report gives the reasoning and calculations supporting use of simple solenoidal magnetic fields to protect the first or inner cavity wall of an inertially confined fusion reactor against the charged particle debris of a thermonuclear microexplosion. Briefly, the reason that simple field geometries are adequate for microexplosions but not for magnetically confined plasmas lies in the shorter plasma time and space of confinement needed. In fact for inertial confinement the magnetic field need not even confine the exploding plasma debris, but need only decelerate or deflect it sufficiently to prevent wall damage. However, our compendious calculations here indicate actual cylindrical confinement, thus protecting the first wall for times well beyond a plasma recoil back toward the axis of the cylindrical reaction cavity.

Our geometry is the simplest, a microexplosion occuring on the axis of a long solenoid, see Fig. 1. A long solenoid is needless in practice, however, for even a single coil may suffice depending on the physics and the geometry of the reactor. Indeed such a coil is both more economical and has more stable convex curved magnetic lines of force axially than the linear lines of a long solenoid. The first wall is taken to be a good conductor, a subject we return to below. If one imagines the poles of the spherically expanding debris to lie on the axis of the cylinder, then the equator of the debris will hit the first wall first (as seen touching the first wall in Fig. 1). The equatorial sector of the debris, because travelling normal to the wall and having the shortest (hence most dense at collision) path to the wall, presents the most severe test of wall protection. In our calculations below we examine the equatorial impacts as a worst case.

As in all magnetic confinement, the critical questions are:

1. Is there instability?

and

2. Do the instabilities that may develop permit energetic plasma penetration to the wall, in this case before general plasma rebound from the compressed magnetic field?

The answers to these questions are, for two numerical cases of interest in laser driven fusion:

1. Negative in part. There is actual stability early in the microexplosion.
2. Negative.

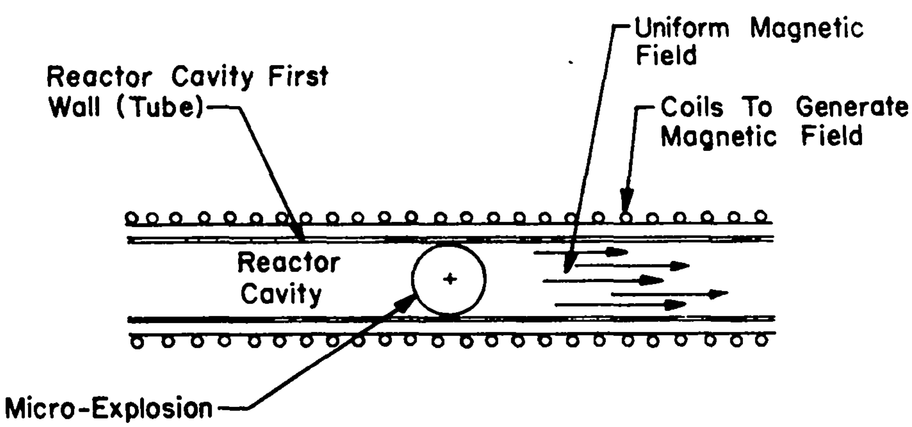

Fig. 1. Magnetically protected cavity wall.

We support these conclusions with three overall calculations for average, but worse case average, microexplosion parameters.

Full, detailed calculations, which unfortunately are extremely lengthy, should certainly be undertaken anticipating any hardware designs. However, the indications available to us presently are that the calculations below are in fact an upper bound to the rates of growth of instabilities.

The three calculations are:

1. Differential Larmor radius stabilization of otherwise weakly unstable confined plasmas.
2. Flute instability growth[2,3] during debris plasma expansion by a method suggested by W. Riesenfeld.[4]
3. Flute instability criteria for conducting plasma shell expanding into an ambient uniform vacuum magnetic field by a method suggested by Poukey.[5]

In addition we present formulas for determining the required initial magnetic field and for determining the wall and plasma skin depths as well as a demonstration of the ambipolar nature of the plasma expansion.

II. REFERENCE MICROEXPLOSION AND DEBRIS PARAMETERS

In order to furnish numerical results for a typical laser-fusion pellet, (for a pure DT pellet see Ref. 6), we adopt the following debris parameters.

Assume that the pellet material consists entirely of lead, weighing 0.253 g, with an asymtotic kinetic energy of $E_0 = 16.17$ MJ, a particle energy of 137.1 keV, a constant charge state 2, and a velocity of 3.57×10^7 cm/s. The number of atoms is 7.36×10^{20}. Initially the lead is at a density $\rho_0 = 0.0535$ g/cm^3 in a shell of outer radius $R_0 = 1.116$ cm. We postulate that the debris expands as a shell with density variation inversely proportional to area thus:

$$\rho = \rho_0 (R_0/R)^2 , \qquad (1)$$

where ρ_0 is the density at the inital outer radius R_0. The first wall cylindrical radius is R_c. This brutal simplification of an otherwise fascinatingly complicated problem is necessary to complete these calculations in a reasonable time. The simplifications are, however, upper bound, or "worst case," approximations. The confinement of the plasma into a spherical shell and its r^{-2} dependence (rather than r^{-3} for instance) provides the highest plasma pressure against the magnetic field and therefore should be worse than the actual distributed plasma. The original charge of the lead ions (at 1 cm) is about 42 which, with expansion and cooling, will eventually drop to near zero. Plasma interaction with the magnetic field will cause currents to flow in the plasma and thereby prevent expansion which in turn will heat the plasma and delay recombination. Our calculations do not depend on the charge to any significant degree unless the charge is very small. See the next section on skin depths. The asymtotic kinetic energy is that kinetic energy that the plasma has at large radii in a free expansion in vacuum, (i.e., after all radiation has occurred).

III. DEBRIS CLOSEST APPROACH, SKIN DEPTHS

In general the closest approach of the plasma to the first wall, d, must be greater than, or about the sum of, the skin depths in the plasma and in the conducting spherical shell. There are other "leakages" (e.g., large flute instability growth) that might warrant choice of d larger than skin depths. However, where a conductivity, σ, can be defined, the skin depth δ is:[7]

$$\delta \approx c(\mu f \sigma)^{-1/2}/2\pi \quad , \tag{2}$$

where μ is the magnetic permeability and f^{-1} is the effective rise time of the magnetic pressure pulse. For example, for copper at 300 K a 1 μs rise-time pulse has a skin depth of 0.0141 cm, but at 500°C the conductivity[8] is reduced by a factor of 3.18 so that the copper skin depth is 0.0251 cm. For graphite at 273 K, the skin depth is 0.317 cm (conductivity, 1.25×10^3 mho/cm)and at 500°C the depth is then 0.323 cm (conductivity, 1.205×10^3 mho/cm).[8]

Plasma conductivity is about[7]

$$\sigma \approx [2m/(Z + 1)e^2 \ell n \Lambda](2kT/\pi m)^{3/2} \quad , \tag{3}$$

where m is the electron mass, e the electron charge, Ze the effective charge of the debris, T is the electron temperature, and k is Boltzman's constant.

For $(3/2)kT < 13.6Z^2$ ev,

$$\Lambda = 12\pi N\lambda_D^3 \quad , \tag{4}$$

and for $(3/2)kT > 13.6Z^2$ eV

$$\Lambda = (Ze^2m/\sqrt{3}\hbar kT)(12\pi N\lambda_D^3) \quad , \tag{5}$$

where N is the free electron density, and the Debye length, λ_D, is

$$\lambda_D = \sqrt{kT/4\pi Ne^2} \quad . \tag{6}$$

As noted, the ionization, density, and temperature are higher for debris pushing against a magnetic field than in a free expansion. As an example, we take lead at a temperature of kT = 1 ev, doubly ionized, and of a density of 10^{-7} g/cm^3, or 5.81×10^{14} electrons/cm^3. (Note: we have chosen a more realistic density for skin depth calculations here than our shell calculations which would be about 1.8×10^{-6} g/cm^2 at r = 190 cm and would give rise to a skin depth of only 1.68 cm). Then Eq. (6) gives a Debye length of

$$\lambda_D = 3.08 \times 10^{-5} \text{ cm} \quad . \tag{7}$$

Eq. (4) applies:

$$\Lambda = 642 \quad , \tag{8}$$

then Eq. (3) gives

$$\sigma = 1.53 \times 10^{13}\ s^{-1} \quad . \tag{9}$$

Finally, using $\mu = \mu_0 = 1$, $f = 0.25$ MHz or a rise time of 1 μs we get

$$\delta = 2.44 \quad . \tag{10}$$

Thus for lead impinging on 500°C graphite, one should use for d the sum of the skin depths, namely,

$$d = 2.8\ \text{cm} \quad , \tag{11}$$

or, perhaps conservatively, a somewhat larger number. In our later examples we will use 10 cm.

For larger chamber radii, R, the quantities, B_0, T, N, and Z all decrease so that λ_0 changes little, and $\ell n\Lambda$ varies but slightly, so that roughly

$$\sigma \propto T^{3/2}/(Z + 1) \quad . \tag{12}$$

If the chamber (i.e., R) is large enough, Z << 1, and

$$\delta \propto T^{-3/4} \tag{13}$$

except for interactions with the magnetic field which will delay recombination. In contrast to our stability calculations, if we approximate the expansion by a free expansion into vacuum we will get an upper bound, but definitely not a least upper bound, to δ. Following Zel'dovich and Razier[9] for isentropic expansion of gas into vacuum with the law,

$$p = A\rho^{\gamma} \ , \tag{14}$$

where p is pressure, ρ density, γ the ratio of specific heats, and A a constant, then the flow is self-similar with

$$\rho = \rho_c(1 - r_d^2/R_d^2)^{\gamma-1} \tag{15}$$

$$p = A\rho_c^{\gamma}(1 - r_d^2/R_d^2)^{\gamma/\gamma-1} \ , \tag{16}$$

where r_d is radial position within the debris and R_d the surface radial position of the flow. The central density is given by

$$\rho_c = cM/R_d^3 \ , \tag{17}$$

where M is the total mass and c is a parametric function of γ determined from the relation

$$M = \int_0^{R_d} \rho \ \ dVolume \ . \tag{18}$$

The particle density, n_p, is related to the density by

$$n_p = \rho A_o/M_A \ , \tag{19}$$

where A_o is Avogadro's number and M_A is the atomic mass of the particle. At low density the perfect gas law applies:

$$p = n_p kT = \rho A_o kT/M_A \quad , \tag{20}$$

so that Eqs. (15), (16), (17), and (20) yield

$$kT = \frac{M_A \; A(cM)^{\gamma-1}(1 - r_d^2/R_d^2)}{A_o R_d^{3(\gamma-1)}} \quad , \tag{21}$$

whence Eq. (13) gives

$$\delta \propto R_d^{9(\gamma-1)/4}(1 - r_d^2/R_d^2)^{3/4} \quad . \tag{22}$$

For $\delta << R_d$ and putting $R_d - r_d = \delta$ we get

$$1 - r_d^2/R_d^2 \approx R_d^{-2} \cdot 2\delta R_d = 2\delta/R_d \tag{23}$$

so that

$$\delta \propto R_d^{9(\gamma-1)/7} R_d^{3/7} \text{ , or } R^{9(\gamma-1)/7} R^{3/7} \quad , \tag{24}$$

which gives an exceedingly crude estimate of the variance of δ with chamber radius R based on required maximum excursion $r = R_d$ of the debris to be such that $d << R$ or $R \approx R_d$. Indeed, for r large, γ approaches 5/3 (ionization is frozen in) and so for the plasma skin depth,

$$\delta \propto R^{9/7} \tag{25}$$

very roughly. Of course, the interaction of the magnetic field with the plasma causes currents to flow in the plasma and also restrains the expansion of the plasma thereby increasing the temperature, the free electron density and the charge (T will dominate), thus by Eq. (2), (3), and (4) acting to decrease δ over the estimate (25).

IV. ELECTRIC DECELERATION OF IONS

The skin depths calculated above give the 1/e penetration of an electromagnetic field into a conductor. In a plasma, the electrons, being so extremely light, furnish the high-frequency response to the rapid rise time field pulse. The gyromagnetic radius is

$$a = mv_p c/Ze\ B \quad , \tag{26}$$

where v_p is the velocity component perpendicular to the magnetic induction, B. For electrons, Z = 1 and the mass m is 2.65 x 10^{-6} of that for a lead ion so that the electron gyromagnetic radius is infinitesimal by comparison to that of lead. Thus, the B-field reverses the electrons long before ion reversal and the ions face a decelerating electric field as well as a turning magnetic field. The question arises: how far beyond the electrons can the ions travel? Let the ion density be n_p and the ion charge be Z, then for a separation x in spherical symmetry we have a net electron charge acting as if from the center, r = o, of

$$G = 4\pi r^2 x n_p Z \qquad \text{for} \quad x \ll r \quad , \tag{27}$$

which gives rise to the mean potential energy per ion of

$$E_{pot} = \frac{GZe^2}{r} \qquad x \ll r \text{ (in erg)} \tag{28}$$

or

$$E_{pot} = 4\pi\ r\ x\ n_p Z^2 e^2 \qquad x << r\ (erg)\ . \tag{29}$$

For lead doubly ionized at a radius of two meters and at a density of 10^{-7} g/cm^3 or 2.91×10^{14} Pb/cm^3,

$$E_{pot} = 4.21\ .\ 10^{11}\ x \qquad (V) \tag{30}$$

so that charge separation of even one micron leads to 42 MV restoring potential per lead ion which is far greater than the usual 100 keV or so maximum kinetic energy of the ions. Thus, the skin depths calculated above need not be increased by charge separation effects. Possible instabilities and other effects will increase the skin depths. Also the calculation given by Eqs. (29) and (30) does not exclude a few ions from exceeding the skin-depth distance, but a few ions will result in negligible wall erosion.

V. MAGNETIC FIELD REQUIRED

Determination of the magnetic field needed to protect the first wall of a laser fusion reactor from microexplosion debris is not trivial because the debris comes out with varying masses, charge states, radiation rates, velocities, pressures, shock structures, and temperatures, all but the first of which vary with time, space, relative position, and chamber background gas and its charge state, mass, velocity, temperature, and density. However, one simple, general,and practical calculational approach exists.[6] That approach bypasses much of the usual difficulties by calculating only the equatorial part of the pellet-field interaction - the limiting part for first wall protection - and by performing the calculation via momentum conservation, which bypasses the ionization states, energy states, and temperature of both debris and residual gas. In this method one requires as input either an estimate of the asymptotic kinetic energy, or of the momentum, or of the velocity of the debris. That is, one must run an explosion code or calculation long enough to allow radiation to become negligible or sufficiently known so that it may be compensated for. The principles of this calculation follow. We ignore instabilities for the moment.

We construct a perfectly conducting long cylinder of radius R_c (see Sec III for nonperfection), containing an initial magnetic induction, B_0, uniformly across the interior and parallel to the axis of the cylinder. The debris plasma is taken at first to be perfectly conducting and of outer radius, R, the equatorial part of which compresses the magnetic induction, B, between the debris and the wall according to

$$B = B_0 / \left[1 - (R/R_c)^2\right] . \tag{31}$$

We now bypass the sphere-cylinder geometrical problem by taking the magnetic pressure over the whole debris sphere to be equal to that at the equator,

$$P_m = B^2/8\pi \quad , \qquad \text{(Gaussian units)} \tag{32}$$

with B given by Eq. (31) over the whole debris surface, a worst case.

Let the debris have an initial outward net momentum, p_0, which would be the asymptotic momentum in the absence of magnetic or wall interactions. Note that the presence or absence of chamber gas will not affect the total chamber momentum p_0; only the magnetic field or the wall can do work on the debris and so change the total momentum. The momentum change, Δp, brought about by a force F in ΔR is given by,

$$\Delta p = - F(R) \frac{dt}{dR} \Delta R \quad . \tag{33}$$

Combining Eqs. (31) and (32) and using the area, $4\pi R^2$, F is:

$$F = PA = R^2 B_0^2/2 \left[1 - (R/R_c)^2\right]^2 \quad . \tag{34}$$

As the debris expands it sweeps up chamber gas, if any, and

$$m = m_o + \rho_{gas} \quad (4\pi/3)R^3 \qquad (35)$$

gives the mass increase. This formula approximates the debris to background gas interaction to be short range which is not always correct because high-energy, tenuous debris will penetrate considerably into the gas. Multiplying both sides of Eq. (33) by p = mv, using v = dr/dt, and substituting (34) and (35) gives,

$$p\Delta p = -\, Fm\Delta R \qquad (36)$$

or

$$p\Delta p = -(B_o^2R_c^4/2)\left[R^2/(R_c^2 - R^2)^2\right]\left[m_o + (4\pi\rho_{gas}/3)R^3\right]\Delta R \quad . \qquad (37)$$

Integrating from momentum p at R = 0 to momentum 0 at R = R gives,

$$2\,p^2 = B_o^2R_c^4 \left\{ m_o \left[\frac{R}{\left(R_c^2 - R^2\right)} - \frac{1}{2R_c}\,\ell n\left|\frac{R_c + R}{R_c - R}\right|\right]\right.$$

$$\left. + \frac{4\pi\rho_{gas}}{3}\left[\frac{R^2\left(2R_c^2 - R^2\right)}{R_c^2 - R^2} + 2R_c^2\ell n\left|\frac{R_c^2 - R^2}{R_c^2}\right|\right]\right\} . \qquad (38)$$

If neither ρ_{gas} or B_o is too large, then R approaches R_c and for most designs that is the optimum. Consequently,

$d \equiv R_c - R$ satisfies $d \ll R_c$ and

$$R_c^2 - R^2 = 2dR_c - d^2 \approx 2dR_c \quad .$$

Thus,

$$2p^2 \approx B_o^2 R_c^4 \left\{ m_o \left[\tfrac{(1/2d)}{3} - \tfrac{(1/2R_c)}{2} \ln \left| 2R_c/d \right| \right] \right.$$

$$\left. + (4\pi\rho_{gas}/3) \left[(R_c/2d) + 2R_c \ln 2d/R_c \right] \right\} \qquad d_c \ll R \quad . \qquad (39)$$

For negligible gas density, ρ_{gas}, we can drop the last terms to get

$$4p^2 \approx m_o B_o^2 R_c^4 \left[(1/d) - (1/R_c) \ln (2R_c/d) \right] \qquad d \ll R_c \text{ no gas.} \qquad (40)$$

Thus, first determining an allowable distance, d, of debris approach to the wall from skin depths (see above, Sect III) then having a given total momentum p_o and initial mass m_o in an evacuated chamber of radius R_c, the required magnetic induction is,

$$B_o^2 \approx (4p_o^2 d/m_o R_c^3) \left[R_c - d \ln(2R_c/d) \right]^{-1} \qquad d \ll R_c \text{ no gas.} \qquad (41)$$

Let us introduce a numerical example, suppose our debris asymtotic kinetic energy is 16.17 MJ = 1.617×10^{14} ergs, mass m_o = 0.253 g, the radius R_c = 200 cm and we take d = 10 cm. In terms of the kinetic energy, E_k, Eq. (41) can then be rewritten

$$B_o^2 \approx \left(8E_k d/R_c^3 \right) \left[R_c - d \ln(2R_c/d) \right]^{-1} \qquad d \ll R_c \text{ no gas.} \qquad (42)$$

Substitution gives the magnetic field, B_o = 3.15 kG, required to stop the debris at 10 cm from the wall. Because $d \ll R_c$ and the logarithm is so much weaker than a linear term, B_o roughly scales according to

$$B_o \approx 2\sqrt{2E_k d}\ R^{-2} \quad . \qquad \begin{array}{l} d \ll R_c \\ \text{no gas.} \end{array} \tag{43}$$

The maximum field for $d = R_c - \bar{R} = 10$ cm is then per Eq. (31), $\bar{B} = 32.3$ kG.

VI. COLLECTIVE PLASMA BEHAVIOR

A whole plasma behaves collectively if its minimum dimension exceeds the Debye length,[10] λ_0, given by Eq. (6). The worst case test is at $R = 190$ cm when $Z = 2$, $N = 2.54 \times 10^{16}/\text{cm}^3$, kT (assumed) = 1 eV, and as before all the plasma is put into a shell at this radius of thickness $d_{shell} = 0.303$ cm. Then Eq. (6) gives,

$$\lambda_D = 4.67 \times 10^{-6} \text{ cm} \quad ; \tag{44}$$

clearly the relation

$$\lambda_D \ll d_{shell} \tag{45}$$

holds, and we may take the plasma behavior to be collective.

VII. FINITE LARMOR RADIUS STABILIZATION

Because the Larmor radii of ions and electrons are finite and different, otherwise weakly unstable confined plasmas actually are stable.[1] The different electron and ion Larmor radii can build up a charge separation out of phase with particle drift separation. Because the latter drives the flute instability, the result can be stable oscillation if:

$$(ka_i)^2 > \omega_H/\Omega_i \quad , \tag{46}$$

where k is the wave number, which we have taken as n/R, with n being the number of flutes; a_i is the ion Larmor radius (gyromagnetic), $a_i = m_i v_i c/e_i B$; Ω_i is the ion Larmor angular frequency (cyclotron frequency), $\Omega_i = e_i B/m_i c$; and ω_H is the hydrodynamic growth rate (Taylor instability).

The growth rate for Taylor instability under gravity is:[11]

$$\omega_H{}^2 = kg \, \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \tag{47}$$

for two fluids of density ρ_1 and ρ_2, k here is the wave number of the instability and g is the gravitational acceleration.

A magnetic field behaves as $\rho_1 \equiv 0$, so $\omega_H^2 = kg \rightarrow k(\bar{R}/R^2)(v_{\parallel}{}^2 + 1/2\ v_{\perp}{}^2)$ for equilibrium.[12] Because R is the radius of curvature of the B-field, this radius is identical to our R; $v_{\parallel}$, $v_{\perp}$ are the velocities parallel and perpendicular to the surface. Because our fluid is not in equilibrium, we must add $\ddot{R}$; also, we are confining our study to the equatorial region where $v_{\parallel} \equiv 0$, thus $g \rightarrow \ddot{R} + (\dot{R}^2/2R)$.

Using the instantaneous total energy $E \equiv 1/2\ M\dot{R}^2$,

$$g = \ddot{R} + E/MR \quad ; \tag{48}$$

hence,

$$\omega_H = \sqrt{k\left[\ddot{R} + (E/MR)\right]} \quad . \tag{49}$$

By substituting Eq. (49) into Eq. (46) our stability criterion reduces to:

$$(n/R)^{3/2} \, \frac{2E_i c}{e_i B} > \sqrt{\ddot{R} + (E/MR)} = \sqrt{\ddot{R} + (\dot{R}^2/2R)} \quad , \tag{50}$$

where E_i is the individual ion energy. When this inequality holds we may expect flute stabilization

To examine the flute stabilization criterion, Eq. (50), we need the plasma outer radius as a function of time. Again we make use of the drastically simplifying feature of applying momentum considerations over a whole sphere, using equatorial parameters exclusively, as proposed in Section V. The rate of change of plasma momentum in such a sphere is equal to the restraining force, or to the product of pressure times area, provided by the magnetic field as given in Eqs. (31) and (32) so that,

$$- dp = Fdt = PAdt = -mdv = B_o^2 R_c^2 R^2 dt / \left(R_c^2 - R^2 \right) . \tag{51}$$

This expression is integrable. With initial conditions, $R = 0$, $t = 0$, and $v = v_o$, we obtain,

$$v^2 = v_o^2 - \left(2B_o^2 R_c^2 / m \right) \left[- R + (R_c/2) \; \ell n \left| (R_c + R)/(R_c - R) \right| \right] . \tag{52}$$

If we expand the logarithm and recognize $v = dR/dt$, then

$$(dR/dt) \equiv \dot{R} = v_o \sqrt{1 - \left(2B_o^2 R_c^3 / m v_o \right) \left[(1/3)(R/R_c)^3 + (1/5)(R/R_c)^5 + \ldots \right]} . \tag{53}$$

Although R can approach R_c closely, using only the first term of the series (53) is a good approximation for our purposes, we therefore write

$$\dot{R} = (dR/dt) \approx v_o \sqrt{1 - (R/\overline{R})^3} , \tag{54}$$

where $\overline{R} = R_c - d$ is the maximum permitted value of R. In our numerical example, $\overline{R} = 190$ cm. Note that any deficiency of this approximation occurs only during the last part of its runs; for $R \ll \overline{R}$ it is very good indeed. We could integrate Eq. (54) to find $R = R(t)$, but the result is an elliptic integral, not a very convenient result. However, use of series provides a result of accuracy greater than the limit provided by Eq. (54). We expand in a Taylor series about $t = 0$ when $R(0) = 0$ and $\dot{R}(0) = v_o$. Using Eq. (54) we find (to sixth order in t) that

$$R(t) = v_o t \left[1 - (1/8)(v_o t/\overline{R})^3\right] . \tag{55}$$

(Note: Eq. (54), good to fourth order in R, still limits this expression. However the expression as used below is entirely adequate for our purposes.) We take only the form of Eq. (55), defining G thereby and write,

$$R = v_o t(1 - Gt^3) \quad , \tag{56}$$

G and t to be determined from the final conditions at plasma rebound,

$$\overline{R} = v_o \overline{t}\,(1 - G\overline{t}^3) \tag{57}$$

and

$$\dot{R} = 0 = v_o(1 - 4\,G\overline{t}^3) \quad , \tag{58}$$

whence $G\overline{t}^3 = 1/4$ so that

$$\overline{t} = 4\overline{R}/3v_o \tag{59}$$

and

$$G = (3v_o)^3/4^4\overline{R}^3 \quad . \tag{60}$$

In our example: $\overline{t} = 7.09 \times 10^{-6}$ s and $G = 7.02 \times 10^{14}$ s^{-3}.

For convenience in applying the test, Eq. (50), as a function of position R, we use Eq. (54) to determine

$$\ddot{R} \approx -\left(3v_o^2/2\overline{R}\right)\left(R/\overline{R}\right)^2 \quad ; \tag{61}$$

then substitution of Eqs. (54) and (61) into (50) gives the test,

$$\left(2nE_ic/e_iB_ov_o\right) \cdot \left[1 - (R/R_c^2)\right] > \sqrt{R^3[(3/2\overline{R})(R/\overline{R})^2 + (1/2R) - (1/2R)(R/\overline{R})^3]}. \tag{62}$$

For our parameters, the inequality holds for the worst case, n = 1, and during $R \leq 106$ cm (for which incidently our approximation, Eq. (54) is a very good one). Thus during the early half of debris expansion we have finite Lamor radius stabilization of flute instabilities. We now proceed to show that the instabilities that do develop do not have sufficient time to grow appreciably during the debris plasma expansion.

VIII. FLUTE INSTABILITY GROWTH

We may expect flute irregularities to grow exponentially.[2,3] Thus an initial irregularity amplitude, A_o, will grow according to

$$A = A_o \exp(t/\tau) \quad , \tag{63}$$

or alternatively, according to

$$dA = (A/\tau)\ dt \quad , \tag{64}$$

where[4]

$$\tau \sim 2\pi R/(v_A \sqrt{n}) \tag{65}$$

and the Alfven velocity is

$$v_A = \sqrt{B^2/4\pi\rho} \tag{66}$$

where as before, n is the number of flutes. The asymmetric explosion, $n = 1$, is the worst case. The plasma mass density is ρ. A worst case calculation of the instability is simply done using all parameters at the maximum radius, $\bar{R}$, because then the plasma is at maximum pressure and the magnetic field, $B = 32.3$ kg, is also at maximum. The resulting time constant, τ, is 1.78×10^{-4} sec, far longer than the expansion time, calculated above, of 7.09×10^{-6} s. However it is possible to calculate the growth over the whole expansion, and get thereby a more accurate growth number. We shall not count on the early stability of the first section, but we will allow instability growth over the whole expansion, and also we put $n = 1$, thus providing a worse case calculation.

Using Eqs. (1), (31), (65), and (66) we find, for $n = 1$,

$$(1/\tau) = K \left[1 - (R/R_c)^2\right]^{-1} = K \left[1 + (R/R_c)^2 + (R/R_c)^4 + \ldots\right] \tag{67}$$

$$= K \sum_{j=0}^{\infty} (R/R_c)^{2j} \quad ,$$

where

$$K \equiv B_o/(4\pi \sqrt{\pi\rho_o} R_o) \ . \tag{68}$$

This series is absolutely and uniformly convergent for $R < R_c$ so that Eq. (67) can be integrated term by term and the resulting series converges to the integral of t^{-1}. Equation (56) gives

$$R^2 = v_o^2(t^2 - 2\,Gt^5 + G^2t^8) \quad . \tag{69}$$

whence

$$\int_0^{\bar{t}} R^2 dt = v_o^2\left(\frac{\bar{t}^3}{3} - 2\,\frac{G\bar{t}^6}{6} + \frac{G^2\bar{t}^9}{9}\right) \quad . \tag{70}$$

Similarly for higher even powers of R.

The numerical coefficients are composed of alternating signs times the binomial coefficients $\binom{m}{i}$ in the numerator and divided by $(m + 1 + 3i)$ in the denominator, thus:

$$\int_0^{\bar{t}} R^m dt = v_o^m \sum_{i=0}^{m} (-1)^i \binom{m}{i} (m + 1 + 3i)^{-1} G^i \bar{t}^{(m+1+3i)} \quad . \tag{71}$$

Using Eq. (67):

$$\int_0^{\bar{t}} (1/\tau)dt = K \sum_{j=0} (v_o/R_c)^{2j} \sum_{i=0}^{2j} (-1)^i$$

$$\cdot \binom{2j}{i}(2j + 1 + 3i)^{-1}\, G^i \bar{t}^{(2j+1+3i)} \quad . \tag{72}$$

For our example $(R_c/v_0) = 5.6 \times 10^{-6}$ s and $\bar{t} = 7.09 \times 10^{-6}$ s, so that the convergence of Eq. (72) is miserably slow.

For our example we calculate the first six terms in the j sum of Eq. (72), i.e., to order $2j = 10$, and then we take advantage of the fact that the major contribution to the integral Eq. (71) for large m comes from $R \sim \bar{R}$. At that value $R \sim 3v_0t/4$ by Eqs. (56) and (60). (For early times $R \sim v_0t$). Thus in the integral,

$$I_m \equiv \int_0^{\bar{t}} (R/R_c)^m dt \quad , \tag{73}$$

we make the substitution

$$R = 3\varepsilon_m v_0 t/4 \tag{74}$$

where ε_m has the bounds

$$(4/3) > \varepsilon_m > 1 \quad \text{and} \quad \varepsilon_m \xrightarrow[m \to \infty]{} 1 \quad , \tag{75}$$

and is a small correction to be determined. Note that we cannot substitute for dt in Eq. (73) with Eq. (74). We would then involve $\dot{R}$ which goes to zero in the limit $R \to \bar{R}$ and our approximation would be totally inaccurate. However substituting Eq. (74) for R in Eq. (73), integrating and substituting Eq. (59) in the result yields:

$$I_m = \left[\varepsilon_m^m \bar{t}/(m + 1)\right](\bar{R}/R_c)^m \quad . \tag{76}$$

To determine the accuracy of our approximation (76) we compare KI_m with the exact terms calculated from Eq. (72) to find that ϵ_m ranges from 1.113 for m = 6 to 1.087 for m = 10 and decreases nearly linearly so that a near linear graphical extrapolation of ϵ_m to 1 (at m = 38) is satisfactory. Eq. (76) is then calculated out to 55 terms and added to the exact first six j-terms calculated from Eq. (72) to give

$$\int_0^{\bar{t}} (1/\tau)\, dt \approx 5.15 \times 10^{-5} \tag{77}$$

in our example. Whence via Eq. (63)

$$A/A_0 = 1.0000515 \quad , \tag{78}$$

or flute irregularities grow at most by 0.005% during the whole plasma expansions, even if there were no finite Lamor radius stabilization (See Sect. VII). We conclude that flute instabilities do not have sufficient time to become troublesome in a thermonuclear microexplosion of, or similar to, our parameters.

IX. PLASMA STABILITY OF A SPHERICAL SHELL EXPANDING INTO A LARGE-SCALE MAGNETIC FIELD

We present in this section calculations based on the findings of Poukey[5] who examined the expansion of a highly conducting spherical shell of plasma into a constant uniform magnetic field. The calculations of this section provide an independent evaluation of stability compared to our calculations of Sect. VIII.

Flute-instability growth of a spherical conducting plasma shell expanding into a large vacuum against a magnetic field is given by the usual formulae proportional to the growth term $e^{t/\tau}$ where the time constant, τ, is now:

$$\tau = (2/3)(n\alpha)^{-1/2} \qquad n\alpha \gg 1 \tag{79}$$

$$\tau = \left(\frac{32}{81}\right)^{2/3} (n\alpha)^{-2/3} \qquad n\alpha \gg 1 \quad , \tag{80}$$

n being the number of flutes and

$$\alpha \equiv \frac{B^2 R_o^3}{2 M v_o^2} = \frac{B^2 R_o^3}{4 E_o} \quad \text{(Gaussian units)} \quad . \tag{81}$$

Here B is the (constant) magnetic induction, uniform throughout space; R_0 is the radius of the sphere at t = 0 expanding outward with an initial velocity v_0, a total mass M, and a total initial kinetic energy E_0. As before we take n = 1, since the asymmetry of implosion is most likely a simple off-center (n = 1) type and also n = 1 is a worst case.

In our example E_0 = 16.17 MJ = 1.617 x 10^{14} erg and for a worst case calculation we take B to be the maximum, $\bar{B}$ = 32.3 kG; we then get, for the plasma:

$$n\alpha = 2.24 \times 10^{-6} \ll 1 \quad , \tag{82}$$

and the second time-constant formula, e.g. Eq. (79), yields:

$$\tau = 3.1 \times 10^3 \text{ s} \quad . \tag{83}$$

A time adequately long indeed for all gases to exit a resonably sized chamber, not to speak of flute instability development. In fact, using the time $\bar{t}$ of Eq. (59) for our example, 7.09 x 10^6, the flute amplitude growth

$$A/A_0 = e^{\bar{t}/\tau} = 1 + 2.25 \times 10^{-9} \quad . \tag{84}$$

Thus even if the magnetic field were equal to the maximum over the whole expansion, flute instabilities would grow only by $2 \times 10^{-7}\%$. This calculation should be an upper bound to the flute amplitude growth of our spherical shell expansion beginning at 3.15 kG and ending at 32.3 kG according to the theory of Poukey.

X. CONCLUSIONS

The calculations of this note strongly support by two (and a half) independent approaches the possibility of magnetic protection of a cavity first wall against energetic plasma debris from microexplosions. Our examples here involved heavy, high Z, debris (0.25 g of lead). Similar conclusions for DT debris were reached in Ref. 6. The limitations of these calculations, in addition to those found in the formulae sources (for which see the references), are in the use of average parameters to represent a whole gamut of physical values. However, such averaging is entirely appropriate for a scoping calculation as intended here. Further, one is highly encouraged in the validity of the above conclusion: first, by the fact that the averagings are in the direction of a worst case; second, that the plasma is actually stable over a part of its travel; and third, that the instabilities as calculated by two approaches then develop appreciably only in times much longer than plasma expansion times.

We also conclude that plasma and wall skin depths are generally small; and that the protective magnetic field required, even for a small reaction chamber, is modest.

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